Methods to Improve the Minimization of an Ising Objective Function

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D-Wave Solution Goals

- Simulate quantum mechanical systems (physics/chemistry)

- Find the globally minimum solution of an objective function.

  (Some may only be interested in a minimal solution.)

Objective function: $f(q) = \sum_i a_i + \sum_i \sum_j b_{ij} q_i \, q_j$

Towards the goal of finding the global minimum

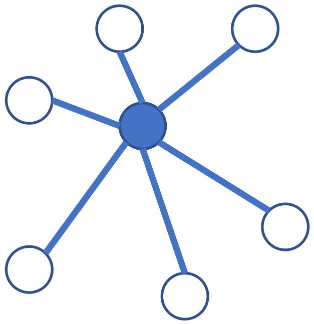
- Patch the values of bad qubits
- Find samples that are a local minimum of the objective function that correspond to each D-Wave sample
  - SQC – single qubit correction
- Find pseudo-tunnels that lead to the global minimum of the objective function
  - MQC – multiple-qubit correction
- Find a more minimal solution for an objective function with higher precision coefficients
  - HPE – high precision enhancement

Bad Qubit Value Patching

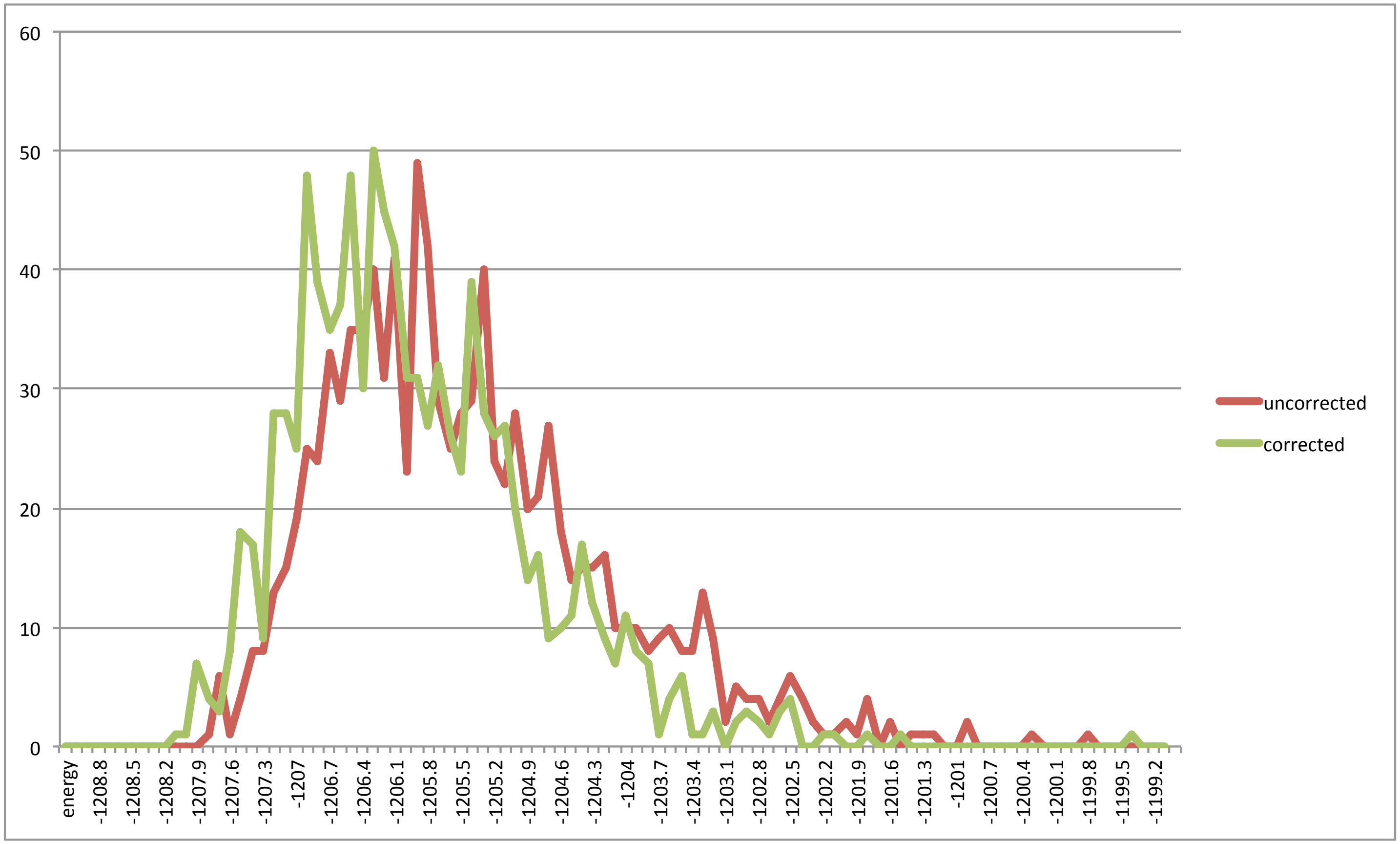
- Bad qubits are found in connected groups (of 1 or more qubits).
  - A connected group is a group of qubits that would have been connected if they were working
- For each sample a new value for the qubits of each bad qubit group is computed and included in the replacement sample.
- Given the values of the good qubits, the new values of a bad qubit group is calculated by exhaustively checking all possible set of values for the qubits of the group.
- The set of values for the bad qubits are chosen as the set of values that gives the lowest value of the objective function.

SQC – Single Qubit Correction

- Each qubit has a region of influence and an influence value associated with the region.
- If one negates the value of a qubits the value of its influence is negated.
- If the influence value is positive and the qubit value is negated, then objective value decreases
- SQC repeatedly looks for qubits that have a positive value and negates them until no more qubit with positive influence can be found
- A local minimum of the objective function has been found.



1000 samples corrected with SQC
(samples generated from a set of random coefficients)



Multi-qubit Correction
MQC

MQC Anchor Problem

Quantum Adiabatic Objective Function

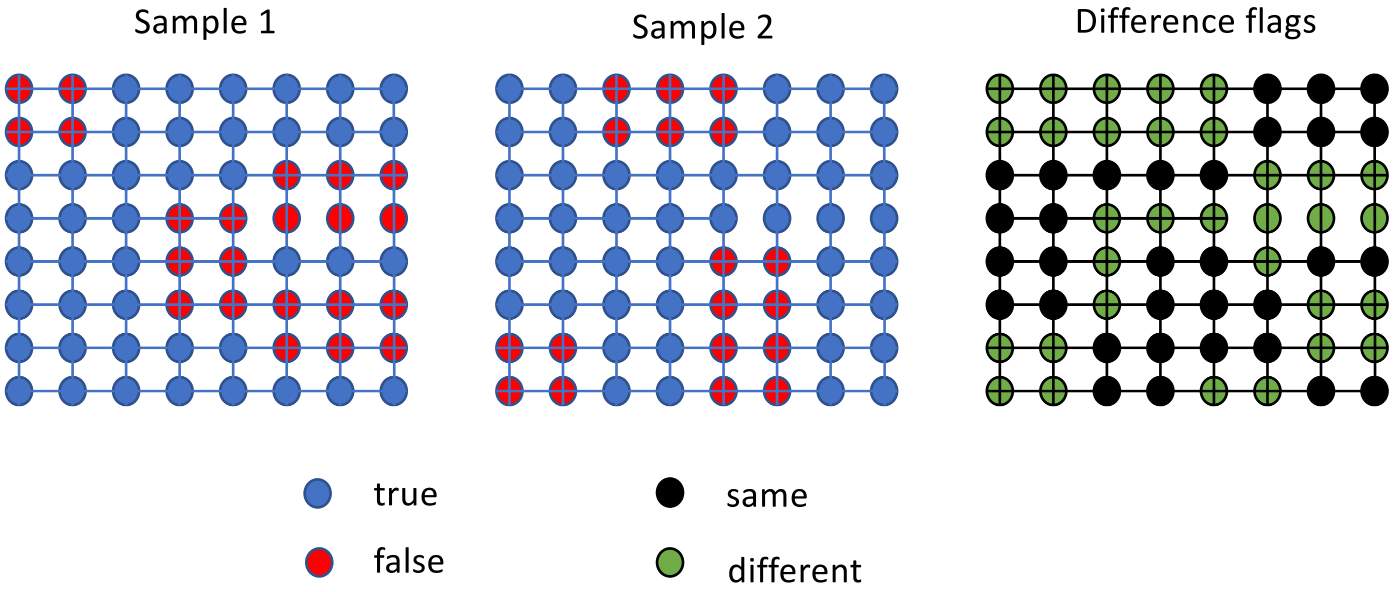
$$Find\ Minimum\ of\colon\ f(q) = \sum_i a_i + \sum_i \sum_j b_{ij} q_i\, q_j$$

D-Wave finds small f(q) :

- Use D-Wave to find hints as what the minimum of F(q) is.

MQC Step 1

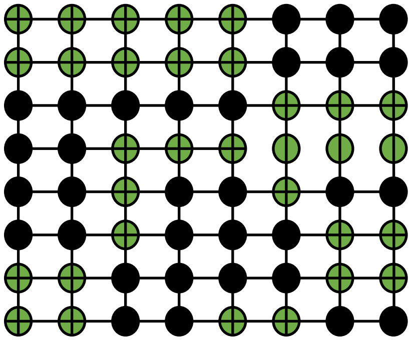
- Compare 2 samples



MQC Step 2

- Find independently connected groups of qubits (tunnels)

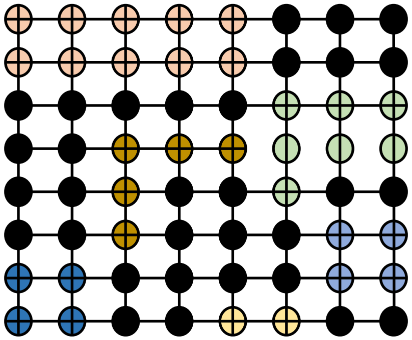
Difference flags



● same

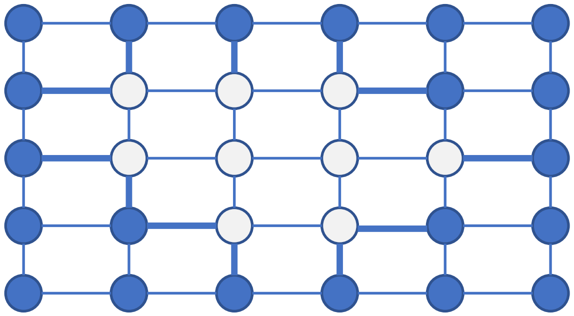
🟢 different

Independent Groups



Transitively connected

MQC Tunnel Influence



Influence

● 0 zero

○ $a_i \; q_i$

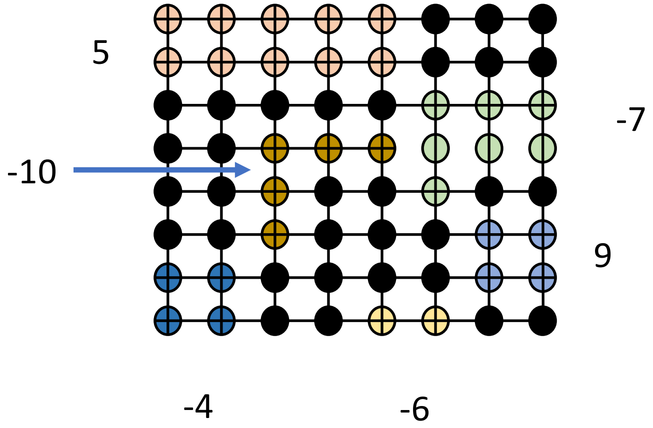
─ 0 zero

━ $b_{ij} \; q_i \; q_j$

MQC Step 3

- Find which tunnels have a positive influence value.

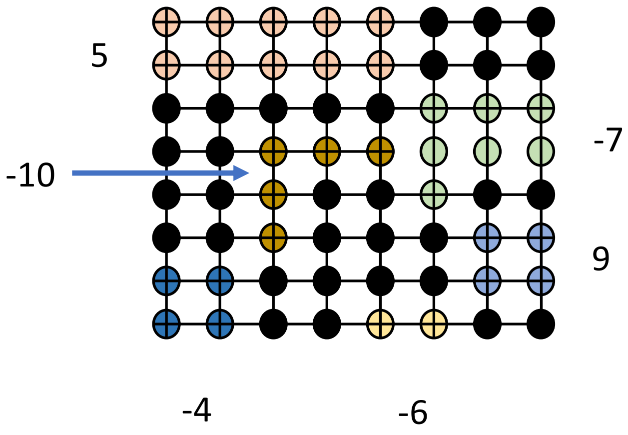
Independent Tunnels



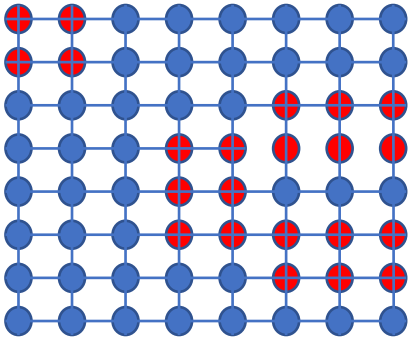
MQC Step 4

- Flip qubits of tunnels with a positive influence value.

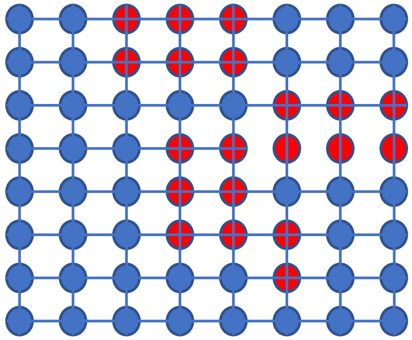
Independent Tunnels



Sample 1



New Sample



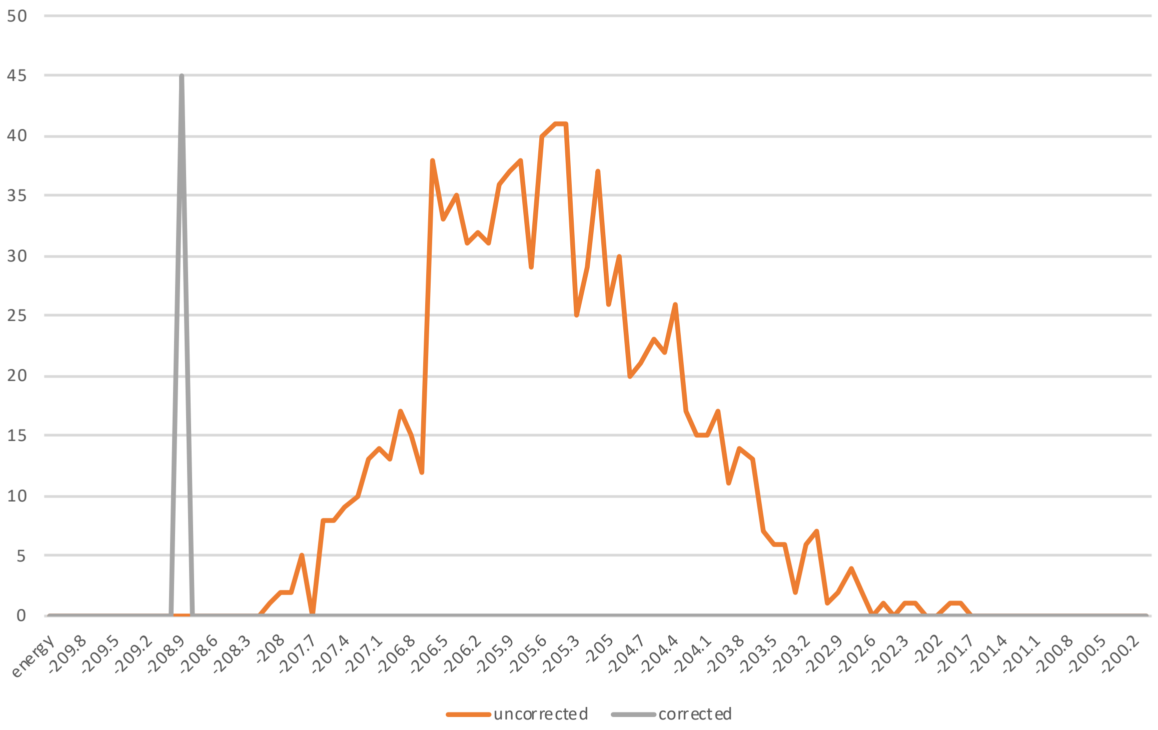
The new sample has a lower value than either sample 1 or 2.

MQC Final Reduction
(for n samples)

- Pair up the n samples and form n/2 new samples
- Pair up the n/2 new samples and form n/4 newer samples
- Repeat until only one sample is left
- This final sample is equal to or less than the value of any of the initial samples.

1000 samples corrected with MQC

(samples generated from a set of random coefficients)



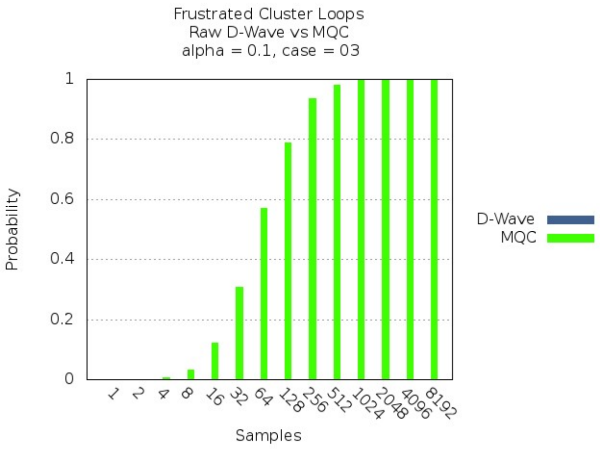
Multi-qubit Correction (MQC)

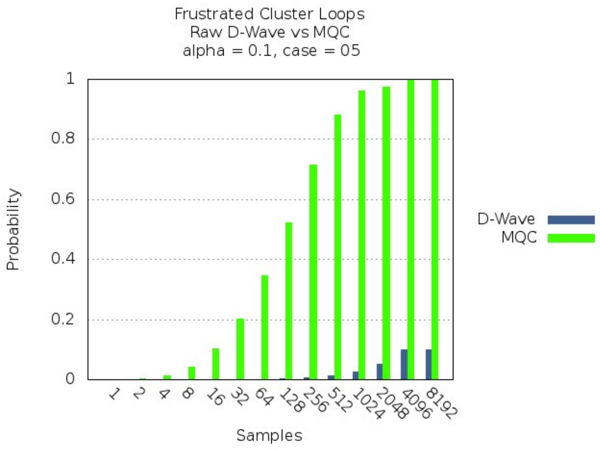
- MQC
  - Finds groups of independent groups of qubits (tunnels) that can potentially reduce the value of the objective function.
  - Determines the influence of each tunnel.
  - Finds a lower value of the objective function through lower energy tunnels.
- Use cases
  - Random objective function coefficients
    - Over 1000 cases improved the objective function value in more than 99% of the cases
  - Boltzmann machine
    - Was trained more rapidly than using the D-Wave without MQC
  - Virtual Qubits (qubit chains)
    - Verified that for all case, MQC found the global minimum

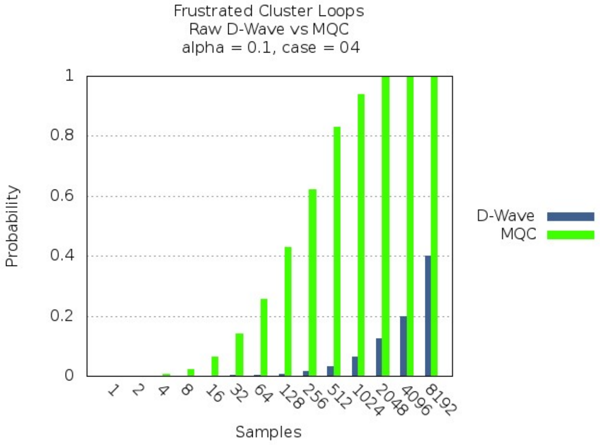
Frustrated Cluster Loops (FCL)

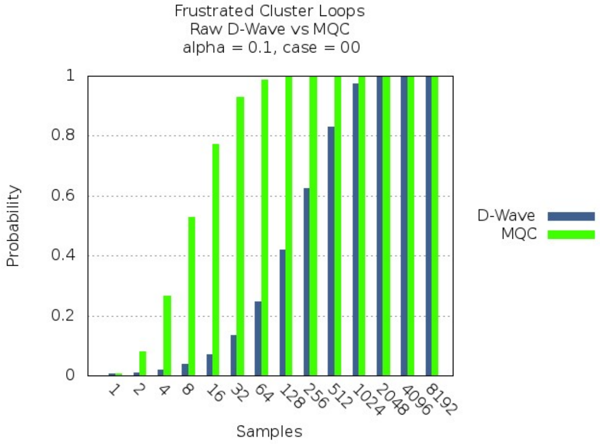
- An FCL is a set of qubits and corresponding coupler that form a closed loop.
- The qubit coefficients are given a value zero.
- The all couplers are given a value of -1, except one which is given a value of 1.
- Multiple loops are randomly generated (α*L_c).
- They are added to give the final set of qubit and coupler coefficients.
- L_c Refers to a partial virtual D-Wave of size c*c cells or c*c*8 qubits

Comparison of Raw vs MQC FCL results

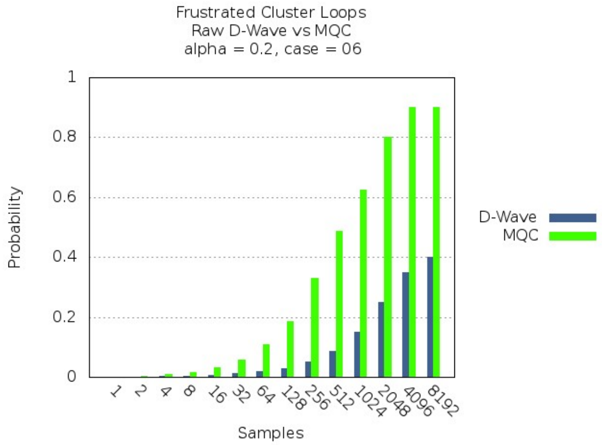


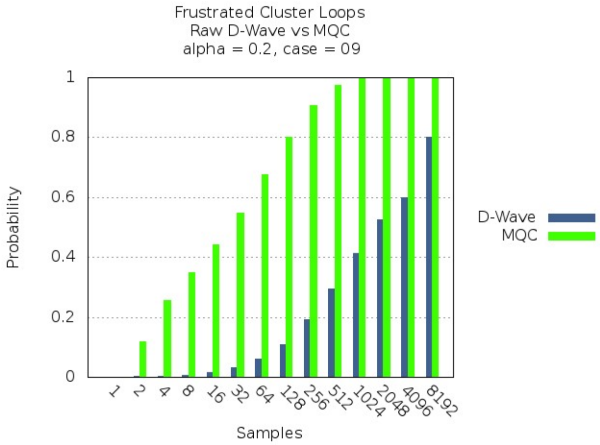


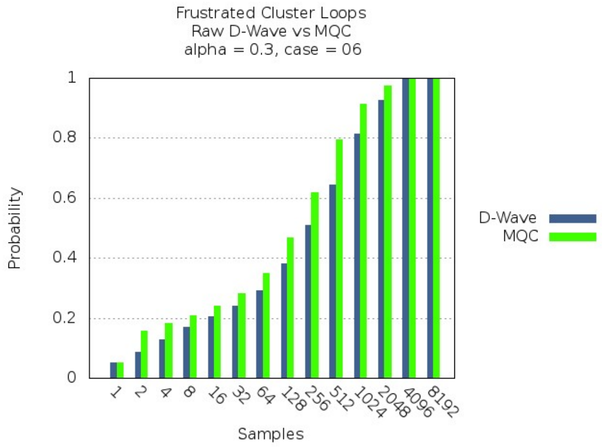


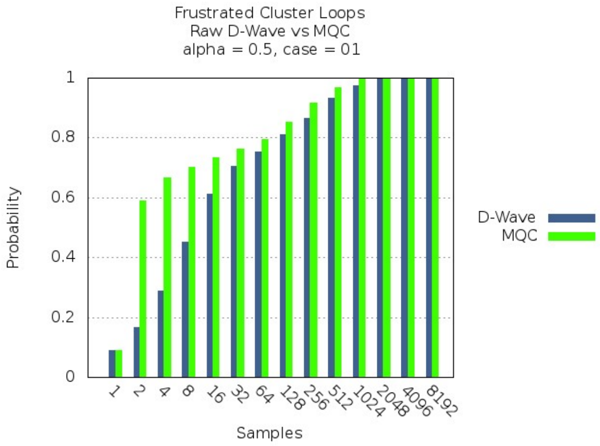


Comparison of Raw vs MQC FCL results (more)









Distribution of the no. of cases solved with no. of samples

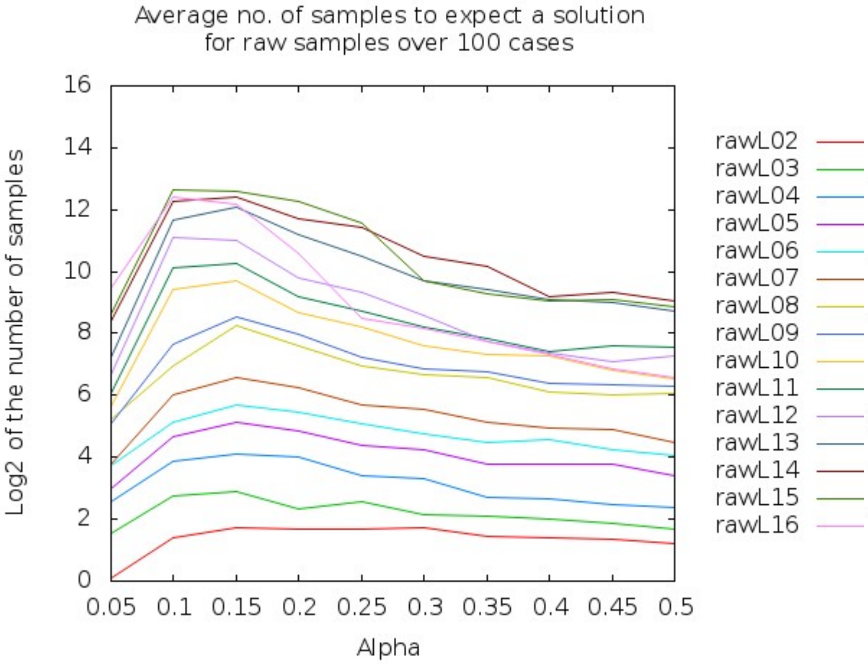
Raw samples for L_{16} and unlimited loop overlap

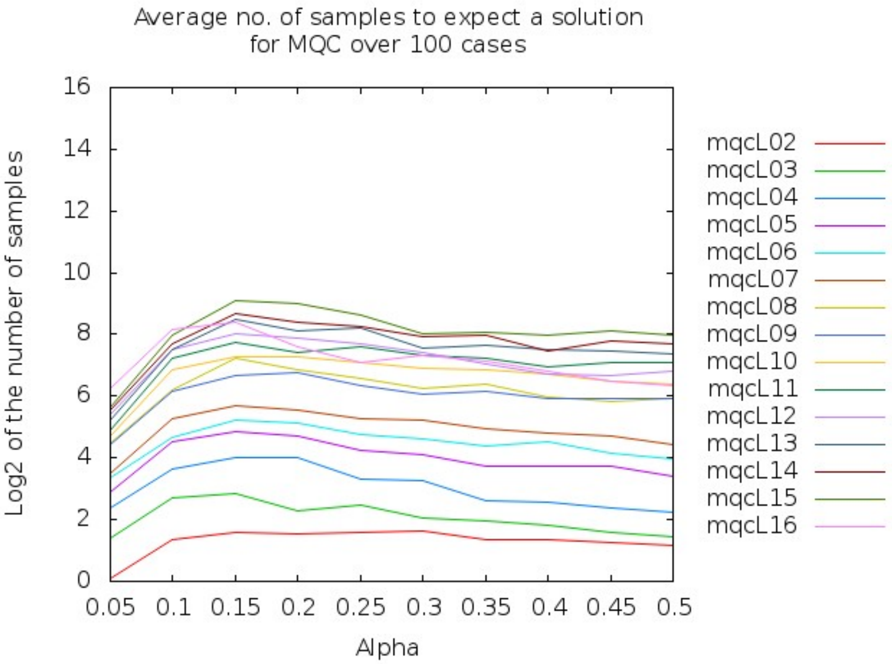
No. of samples	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	unsolved	
Alpha = 0.05 --	0	0	0	0	0	4	5	14	32	20	16	5	2	2	0	0
Alpha = 0.10 --	0	0	0	0	0	0	0	0	0	0	7	8	12	28	0	45
Alpha = 0.15 --	0	0	0	0	0	0	0	0	0	0	14	20	20	30	0	16
Alpha = 0.20 --	0	0	0	0	0	0	0	4	13	28	30	10	6	7	0	2
Alpha = 0.25 --	0	0	0	0	0	0	4	22	43	24	5	2	0	0	0	0
Alpha = 0.30 --	0	0	0	0	0	0	4	30	45	17	4	0	0	0	0	0
Alpha = 0.35 --	0	0	0	0	0	1	15	42	28	12	2	0	0	0	0	0
Alpha = 0.40 --	0	0	0	0	0	3	26	47	18	5	1	0	0	0	0	0
Alpha = 0.45 --	0	0	0	0	1	9	38	38	13	1	0	0	0	0	0	0
Alpha = 0.50 --	0	0	0	0	3	17	36	35	9	0	0	0	0	0	0	0

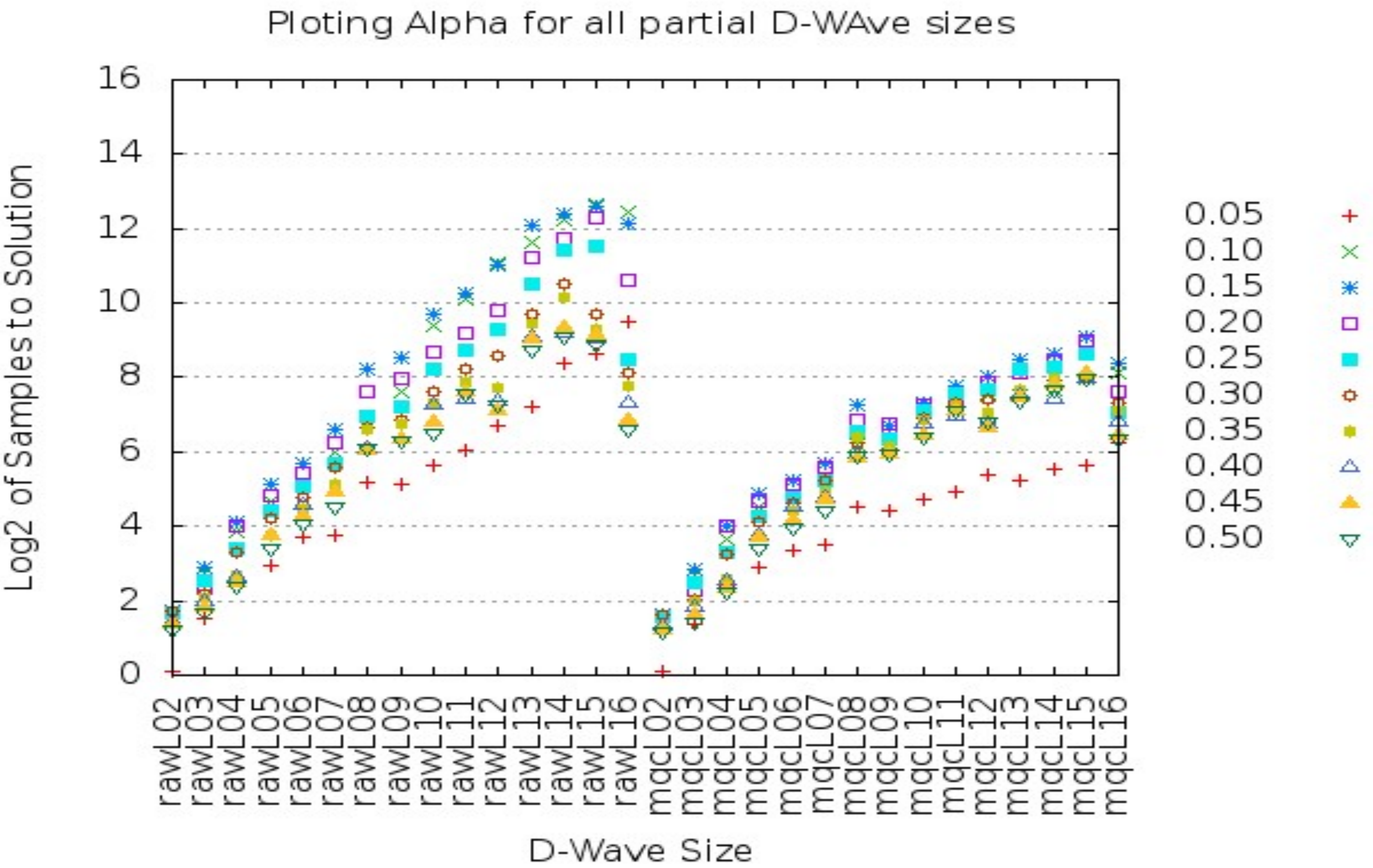
MQC samples for L_{16} and unlimited loop overlap

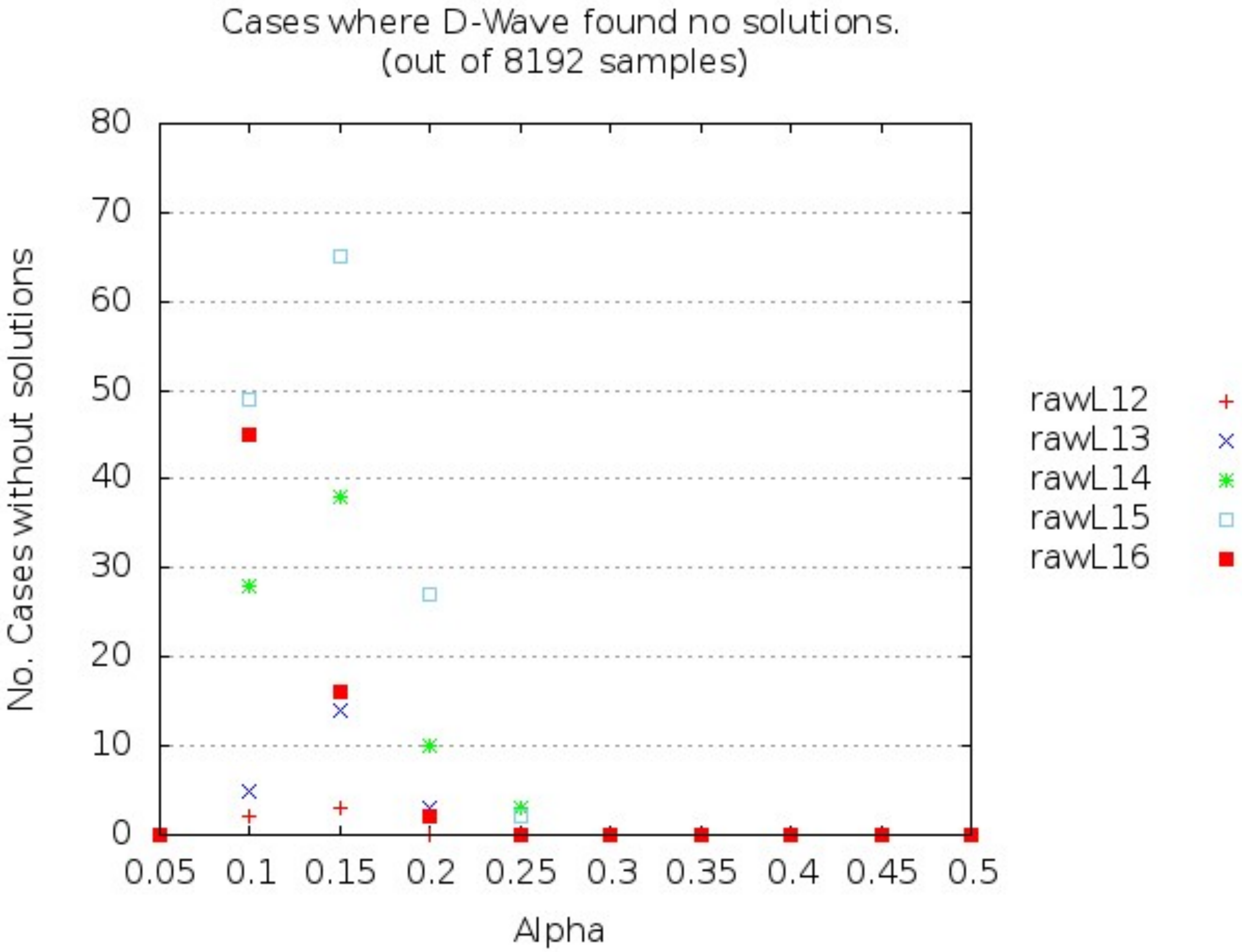
No. of samples	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	unsolved	
Alpha = 0.05 --	0	0	0	0	3	19	54	21	3	0	0	0	0	0	0	0
Alpha = 0.10 --	0	0	0	0	0	0	2	28	48	20	2	0	0	0	0	0
Alpha = 0.15 --	0	0	0	0	0	0	0	18	51	27	4	0	0	0	0	0
Alpha = 0.20 --	0	0	0	0	0	0	10	53	30	5	2	0	0	0	0	0
Alpha = 0.25 --	0	0	0	0	0	1	28	53	17	1	0	0	0	0	0	0
Alpha = 0.30 --	0	0	0	0	0	0	23	50	23	4	0	0	0	0	0	0
Alpha = 0.35 --	0	0	0	0	0	2	36	43	16	2	1	0	0	0	0	0
Alpha = 0.40 --	0	0	0	0	1	10	40	36	11	2	0	0	0	0	0	0
Alpha = 0.45 --	0	0	0	0	3	20	38	33	6	0	0	0	0	0	0	0
Alpha = 0.50 --	0	0	0	0	5	26	34	31	4	0	0	0	0	0	0	0

The Average of samples that are need to solve a case






Ploting Alpha for all partial D-WAve sizes
Log2 of Samples to Solution
D-Wave Size
0
2
4
6
8
10
12
14
16
rawL02
rawL03
rawL04
rawL05
rawL06
rawL07
rawL08
rawL09
rawL10
rawL11
rawL12
rawL13
rawL14
rawL15
rawL16
mqcL02
mqcL03
mqcL04
mqcL05
mqcL06
mqcL07
mqcL08
mqcL09
mqcL10
mqcL11
mqcL12
mqcL13
mqcL14
mqcL15
mqcL16
0.05 +
0.10 ×
0.15 ✳
0.20 □
0.25 ■
0.30 ○
0.35 ■
0.40 △
0.45 ▲
0.50 ▽


Cases where D-Wave found no solutions.
(out of 8192 samples)
No. Cases without solutions
0
10
20
30
40
50
60
70
80
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Alpha
rawL12 +
rawL13 ×
rawL14 ✳
rawL15 □
rawL16 ■

Enhanced D-Wave Coefficient Precision (HPE-Higher Precision Enhancement)

Base Expression: $F(q) = \sum_i a_i q_i + \sum_{ij} b_{ij} q_i q_j$

Version Expressions: $F_k(q) = \sum_i c_k a_i q_i + \sum_{ij} c_k b_{ij} q_i q_j$

$$d = \max_{ij}\left(abs(a_i), abs(b_{ij})\right)$$

$$c_0 = \frac{1}{8 * d}$$

$$c_k = c_{k-1}\sqrt{2} = c_0\left(\sqrt{2}\right)^k$$

$Note$: $F(q)$ and $F_k(q)$ have the same minima.

HPE-Higher Precision Enhancement Method

- Step 1: D-Wave generates samples for each version of objective function.
  - F^k➔S^k where $k = 0 \ldots k_{max}$
- Step 2: Create sample groups.
  - $s_h^k \in S^k$ where $\mathrm{h} = 0 \ldots samples$
  - $s_h^k \in T^h$
- Step 3: Reduce samples groups to single samples.
  - $MQC\left(T^h, F\right) = t_h$
- Step 4: Reduce single samples to a single sample group.
  - $t_h \in H$
  - $MQC(H, F) = h$
- h is the result sample from HPE for F

Enhanced D-Wave Coefficient Precision Version constants

- c_k is intended to scale the coefficients, a_i and b_{ij}, from all being near zero for c_0 to all being either 1 or -1 for c_{max}
- As k increments from 0 to k_{max} , c_k increases by factors of $\sqrt{2}$ which is in effect a half a bit of resolution
- d = the maximum of the absolute values, a_i and b_{ij} of F

HPE Method

1) Create multiple versions (k=20) of the ‘Base expression’.
2) Use D-Wave to solve each version (1000 result samples each).
3) Use MQC to collapse the sample results into a single result.

Bexp	Vexp	Cases	Samples	Raw<HPE	MQC<HPE	MQC=HPE	HPE<MQC
9	3	1000	1000	9	775	106	119
9	3	100	10000	1	56	28	16
17	3	100	10000	0	59	30	11
25	3	100	10000	0	56	34	10
33	3	100	10000	0	52	38	10

Base Precision(Bexp) vs. Pseudo-Hardware(Vexp) Test cases.

Bexp	Vexp	Cases	Samples	Raw<HPE	MQC<HPE	MQC=HPE	HPE<MQC
9	9	100	1000	0	0	37	63
17	9	100	1000	0	0	48	52
25	9	100	1000	0	0	45	55
33	9	100	1000	0	0	45	55
41	9	100	1000	0	0	44	56
49	9	100	1000	0	0	48	52

Base Precision(Bexp) vs. D-Wave(Vexp) Test cases.

Bexp	Vexp	Cases	Samples	Raw<HPE	MQC<HPE	MQC=HPE	HPE<MQC
dbl	dbl	100	1000	0	0	39	61

Unconstrained Base Precision vs. Unconstrained D-Wave Test cases.

References

- 2017 J. E. Dorband, "Improving the Accuracy of an Adiabatic Quantum Computer", eprint arXiv:1705.01942, May 2017.
- 2018 J. E. Dorband, "A Method of Finding a Lower Energy Solution to a QUBO/Ising Objective Function", eprint arXiv:1801.04849, Jan. 2018.
- 2018 J. E. Dorband, "Extending the D-Wave with support for Higher Precision Coefficients", eprint arXiv:1807.05244, July 2018.

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  - ORNL for some of their time on the QPU at Burnaby , BC
  - NASA for time on the QPU at NASA Ames

Questions?